First Law of Black Rings Thermodynamics in Higher Dimensional Dilaton Gravity with $p+1$ Strength Forms

Marek Rogatko
Institute of Physics
Maria Curie-Sklodowska University
20-031 Lublin, pl.Marii Curie-Sklodowskiej 1, Poland
rogat@tytan.umcs.lublin.pl
rogat@kft.umcs.lublin.pl

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We derive the first law of black rings thermodynamics in n-dimensional Einstein dilaton gravity with additional $(p+1)$-form field strength being the simplest generalization of five-dimensional theory containing a stationary black ring solution with dipole charge. It was done by means of choosing any cross section of the event horizon to the future of the bifurcation surface.

I. INTRODUCTION

Recently there has been a great resurgence of interests in n-dimensional generalization of black holes motivated by various attempts of constructing the unified theories. As far as the static n-dimensional black holes is concerned the uniqueness theorem for them is quite well established [1]. But for stationary axisymmetric ones the problem is much more complicated. In Ref. [2] it was shown that even in five-dimensional spacetime there is the so-called *black ring* solution possessing $S^2 \times S^1$ topology of the event horizon and having the same mass and angular momentum as spherical five-dimensional stationary axisymmetric black hole. On the contrary, when one assumes the spherical topology of the horizon S^3 the uniqueness proof can be conducted (see Ref. [3] for the vacuum case and Ref. [4] for the stationary axisymmetric self-gravitating σ-model). As was remarked in [5] due to *the vast number of the ways of forming of these objects, one can take the view that the spirit of no-hair theorem is preserved*.

The black ring solution was vastly treated in literature, i.e., the solution possessing the electric and magnetic dipole charge was found [6,7], static black ring solution in five-dimensional Einstein-Maxwell-dilaton gravity was presented in [8] and systematically derived in [9] both in asymptotically flat and non-asymptotically flat case. Recently, the rotating non-asymptotically flat black ring solution in charged dilaton gravity was given [10]. Also the supersymmetric black ring solutions were discovered [11].

As far as the black hole thermodynamics is concerned, Wald [12] referred two versions of the black hole thermodynamics. The first one, the so-called *equilibrium state version* was treated for the first time in the seminal paper of Bardeen, Carter and Hawking [13]. In this attitude the linear perturbations of a stationary electrovac black hole to another stationary black hole were taken into account. In Ref. [14] arbitrary asymptotically flat perturbations of a stationary black hole were considered. The first law of black hole thermodynamics valid for an arbitrary diffeomorphism invariant Lagrangian with metric and matter fields possessing stationary and axisymmetric black hole solutions was given in Refs. [15]- [18], while the higher curvature terms and higher derivative terms in the metric were considered in [19]. The case when the Lagrangian is an arbitrary function of metric, Ricci tensor and a scalar field

was elaborated in Ref. [20]. Then, the case of a charged and rotating black hole where fields were not smooth through the event horizon was treated in [21].

On the other hand, the *physical process* version of the first law of black hole thermodynamics was realized by changing a stationary black hole by some infinitesimal physical process, e.g., when matter was thrown into black hole was considered. Assuming that the black hole eventually settles down to a stationary state we are able to find the changes of black hole's parameters and establish the first law of black hole mechanics. The *physical process* version of the first law of black hole thermodynamics in Einstein theory was proved in Ref. [12] and generalized for Einstein-Maxwell (EM) black holes in Ref. [22]. For Einstein-Maxwell axion-dilaton (EMAD) gravity black holes it was derived in Ref. [23].

The first law of black hole thermodynamics was also intensively studied in the case of n-dimensional black holes. The *equilibrium state* version was elaborated in Ref. [24] under the assumption of spherical topology of black holes. Some of the works assume that four-dimensional black hole uniqueness theorem extends to higher dimensional case [25]. The *physical process* of the first law of black hole thermodynamics in n-dimension was treated in Ref. [26].

As far as the black ring first law of mechanics is concerned, the general form of this law was established in Ref. [27], using the notion of bifurcate Killing horizons and taking into account dipole charges. The *physical process* version of the first law of thermodynamics in the higher dimensional gravity containing $(p+1)$-form field strength and dilaton fields which constitutes the simplest generalization of five-dimensional one, which in turn contains stationary black ring solution with dipole charge [28], was given in Ref. [29].

In our paper we shall derive the first law of black ring mechanics choosing an arbitrary cross section of the event horizon to the future of the bifurcation surface, contrary to the previous derivations based on taking into account the bifurcation surface as the boundary of the hypersurface extending to spatial infinity. This attitude enables one to treat fields which are not necessarily smooth through the horizon [21], i.e., one requires only that the pullback of the fields under considerations in the future of the bifurcation surface be smooth.

II. THE FIRST LAW OF BLACK RING MECHANICS

We shall consider the simplest higher dimensional generalization of of the five-dimensional theory with three-form field that admits stationary black ring solutions. Namely, it will be subject to the relation

$$\mathbf{L} = \epsilon\bigg({}^{(n)}R - \frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi - \frac{1}{2(p+1)!}e^{-\alpha\phi}H_{\mu_1\dots\mu_{p+1}}H^{\mu_1\dots\mu_{p+1}}\bigg), \tag{1}$$

where ϵ is the n-dimensional volume element, ϕ constitutes the dilaton field while $H_{\mu_1\dots\mu_{p+1}} = (p+1)!\nabla_{[\mu_1}B_{\mu_2\dots\mu_{p+1}]}$ is $(p+1)$-form field strength.

The symplectic $(n-1)$-form $\Theta_{j_1\dots j_{n-1}}[\psi_\alpha, \delta\psi_\alpha]$, was calculated in [29]. It yields

$$\Theta_{j_1\dots j_{n-1}}[\psi_\alpha, \delta\psi_\alpha] = \epsilon_{\mu j_1\dots j_{n-1}}\bigg[\omega^\mu - e^{-\alpha\phi}H^{\mu\nu_2\dots\nu_{p+1}}\ \delta B_{\nu_2\dots\nu_{p+1}} - \nabla^\mu\phi\ \delta\phi\bigg], \tag{2}$$

where $\omega_\mu = \nabla^\alpha\delta g_{\alpha\mu} - \nabla_\mu\delta g_\beta{}^\beta$. For brevity, we denote fields in the underlying theory by ψ_α, while their variations by $\delta\psi_\alpha$. The Noether charges are as follows:

$$Q^{GR}_{j_1 \dots j_{n-2}}(\xi) = -\epsilon_{j_1 \dots j_{n-2} ab} \nabla^a \xi^b, \tag{3}$$

while $Q^B_{j_1 \dots j_{n-2}}$ has the following form:

$$Q^B_{j_1 \dots j_{n-2}}(\xi) = \frac{p}{(p+1)!} \epsilon_{m\alpha j_1 \dots j_{n-1}} \; \xi^d \; B_{d\alpha_3 \dots \alpha_{p+1}} \; e^{-\alpha\phi} H^{m\alpha\alpha_3 \dots \alpha_{p+1}}. \tag{4}$$

Let us consider the case when the spacetime satisfies asymptotic conditions at infinity and the Killing vector ξ^μ guarantees an asymptotic symmetry. Then, there exists a conserved quantity H_ξ associated with the Killing vector fields under consideration [30], which implies

$$\delta H_\xi = \int_\infty \biggl(\bar{\delta} Q(\xi) - \xi \Theta \biggr), \tag{5}$$

where $\bar{\delta}$ denotes the variation which has no effect on ξ_α since the Killing vector field is treated as a fixed background and it ought not to be varied in expression (5).
For the case when a hypersurface Σ extends to infinity and has an inner boundary $\partial\Sigma$, and moreover ξ_α is a symmetry of all dynamical fields as well as ψ_α and $\delta\psi_\alpha$ fulfill the linearized equations of motion, then it follows [16] that the integral over infinity can be changed into the inner boundary one. In our consideration we shall confine our attention to stationary and axisymmetric black ring so the Killing vector field is of the form

$$\xi^\mu = t^\mu + \sum_i \Omega_{(i)} \phi^{\mu(i)}, \tag{6}$$

where $\phi^{\mu(i)}$ are the Killing vectors responsible for the rotation in the adequate directions. As we take ξ^α to be an asymptotical time translation t^α, we obtain from Eq.(5) the variation of canonical energy $\delta\mathcal{E}$ and taking into account $\phi^{\mu(i)}$ we get variations of the adequate canonical angular momentum $\delta\mathcal{J}_{(i)}$.

From this stage on, we shall take into account an asymptotically hypersurface Σ terminating on the portion of the event horizon $\mathcal{H}$ to the future of the bifurcation surface. By $S_\mathcal{H}$ we denote the cross section of the event horizon which constitutes the inner boundary of the hypersurface Σ. We take into account a variation between two neighbouring states of black rings. When one compares two slightly different solutions there is a freedom in which points they can be chosen to correspond. We choose this freedom to make $S_\mathcal{H}$ the same of the two solutions (freedom of the general coordinate transformation) as well as the null vector remains normal to it. The stationarity and axisymmetricity described by the Killing vector fields t^α and $\phi^{\alpha(i)}$ will also be conserved. Thus, the perturbations of the Killing vector fields δt^α and $\delta\phi^{\alpha(i)}$ will be equal to zero, which implies in turn that the corotating Killing vector field is assigned to the expression

$$\delta\xi^\mu = \sum_i \delta\Omega_{(i)} \phi^{\mu(i)}. \tag{7}$$

Let us assume that $(g_{\mu\nu}, B_{\alpha_1 \dots \alpha_p}, \phi)$ are solutions of the equations of motion derived from the Lagrangian (1) and $(\delta g_{\mu\nu},\ \delta B^{\alpha_1 \dots \alpha_p},\ \delta\phi)$ are the linearized perturbations satisfying Eqs. of motion. We require that the pullback of $B_{\alpha_1 \dots \alpha_p}$ to the future of the bifurcation surface be smooth, but not necessarily smooth on it [21]. One assumes further, that $B_{\alpha_1 \dots \alpha_p}$ and $\delta B_{\alpha_1 \dots \alpha_p}$ fall off sufficiently rapid at infinity. Then, those fields do not contribute to the canonical energy and canonical momenta. It implies

$$\alpha\ \delta M - \sum_i \Omega_{(i)}\delta J^{(i)} = \int_{S_{\mathcal{H}}} \bigg(\bar{\delta} Q(\xi) - \xi\Theta \bigg), \tag{8}$$

where $\alpha = \frac{n-3}{n-2}$.

First we calculate the integral over the symplectic $(n-1)$-form bounded with the dilaton field. We express $\epsilon_{\mu a j_1 \dots j_{n-2}}$ by the volume element on $S_{\mathcal{H}}$ and by the vector N^α, the *ingoing* future directed null normal to $S_{\mathcal{H}}$, which is normalized to $N^\alpha \xi_\alpha = -1$ [31]. It gives the expression written as

$$\int_{S_{\mathcal{H}}} \xi^{j_1}\ \Theta^{\phi}_{j_1 \dots j_{n-1}} = \int_{S_{\mathcal{H}}} \epsilon_{j_1 \dots j_{n-2}} N_\alpha \xi^\alpha\ \xi_\mu \nabla^\mu \phi\ \delta\phi = 0, \tag{9}$$

where we used the fact that $\mathcal{L}_\xi \phi = 0$.

Consequently, we turn now our attention to the $(p+1)$-form field. In order to find the integral over the event horizon we take into account the following relation:

$$\begin{aligned} p!\ \mathcal{L}_\xi B_{\alpha_2 \dots \alpha_{p+1}}\ H^{m\alpha_2 \dots \alpha_{p+1}} - \xi^d\ H_{d\alpha_2 \dots \alpha_{p+1}}\ H^{m\alpha_2 \dots \alpha_{p+1}} \\ = p\ p! \nabla_{\alpha_2} \bigg(\xi^d\ B_{d\alpha_3 \dots \alpha_{p+1}} \bigg)\ H^{m\alpha_2 \dots \alpha_{p+1}}. \end{aligned} \tag{10}$$

The first term of the left-hand side of Eq.(10) is equal to zero since the Killing vector field ξ_α constitutes the symmetry of the background solution. Next, let us consider n-dimensional Raychauduri equation written as follows:

$$\frac{d\theta}{d\lambda} = -\frac{\theta^2}{(n-2)} - \sigma_{ij}\sigma^{ij} - R_{\mu\nu}\xi^\mu \xi^\nu, \tag{11}$$

where λ denotes the affine parameter corresponding to vector ξ_α, θ is the expansion and σ_{ij} is shear. Shear and expansion vanish in the stationary background, so the inspection of Eq.(11) yields that $R_{\alpha\beta}\xi^\alpha\xi^\beta \mid_{\mathcal{H}} = 0$, implying the following:

$$\frac{1}{2}\xi^\mu \nabla_\mu \phi\ \xi^\nu \nabla_\nu \phi + \frac{1}{2p!} e^{-\alpha\phi} H_{\mu\mu_2 \dots \mu_{p+1}} H_\nu{}^{\mu_2 \dots \mu_{p+1}} \xi^\mu \xi^\nu \mid_{\mathcal{H}} = 0. \tag{12}$$

Due to the fact that $\mathcal{L}_\xi \phi = 0$, it is easily seen that, $H_\mu{}^{\mu_2 \dots \mu_{p+1}} \xi^\mu = 0$. Because of $H_{\mu\mu_2 \dots \mu_{p+1}} \xi^\mu \xi^{\mu_2} = 0$, and having in mind asymmetry of $H_{\mu_1 \dots \mu_{p+1}}$ one draws a conclusion that $H_{\mu\mu_2 \dots \mu_{p+1}} \xi^\mu \sim \xi_{\mu_2} \dots \xi_{\mu_{p+1}}$. Just the pullback of $H_\mu{}^{\mu_2 \dots \mu_{p+1}} \xi^\mu$ to the event horizon is equal to zero, which in turn constitutes that $\xi^d\ B_{d\alpha_2 \dots \alpha_{p+1}}$ is a closed p-form on the event horizon. Applying the Hodge theorem (see e.g., [32]) it may be rewritten as a sum of an exact and harmonic form. An exact form does not contribute to the above expression because of the Eqs. of motion are satisfied. The harmonic part of $\xi^d\ B_{d\alpha_2 \dots \alpha_{p+1}}$ gives the only contribution. Just, having in mind the duality between homology and cohomology, one can conclude that there is a harmonic form η dual to $n-p-1$ cycle $\mathcal{S}$ in the sense of the equality of the adequate surface integrals. Then, it follows that the surface term will be of the form as $\Phi_l\ q_l$, where Φ_l is the constant relating to the harmonic part of $\xi^d\ B_{d\alpha_2 \dots \alpha_{p+1}}$ and q_l is a local charge [27]. These considerations allow one to write down the following:

$$\int_{S_{\mathcal{H}}} Q^B_{j_1 \dots j_{n-2}}(\xi) = \Phi_l\ q_l. \tag{13}$$

Let us compute the variation $\bar{\delta}$. It reduces to

$$\bar{\delta} \int_{S_{\mathcal{H}}} Q^B_{j_1 \dots j_{n-2}}(\xi) = \delta \int_{S_{\mathcal{H}}} Q^B_{j_1 \dots j_{n-2}}(\xi) - \int_{S_{\mathcal{H}}} Q^B_{j_1 \dots j_{n-2}}(\delta\xi) = \delta\biggl(\Phi_l \ q_l\biggr) \tag{14}$$
$$- \frac{p}{(p+1)!} \int_{S_{\mathcal{H}}} \sum_i \delta\Omega_{(i)} \phi^{\mu(i)} B_{\mu\alpha_3 \dots \alpha_{p+1}} \epsilon_{m\alpha j_1 \dots j_{n-2}} \ e^{-\alpha\phi} \ H^{m\alpha\alpha_3 \dots \alpha_{p+1}}.$$

As immediate consequences of the above expressions one has

$$\delta\Phi_l \ q_l = \frac{p}{(p+1)!} \int_{S_{\mathcal{H}}} \sum_i \delta\Omega_{(i)} \phi^{\mu(i)} B_{\mu\alpha_3 \dots \alpha_{p+1}} \epsilon_{m\alpha j_1 \dots j_{n-2}} \ e^{-\alpha\phi} \ H^{m\alpha\alpha_3 \dots \alpha_{p+1}} \tag{15}$$
$$+ \frac{p}{(p+1)!} \int_{S_{\mathcal{H}}} \xi^d \ \delta B_{d\alpha_3 \dots \alpha_{p+1}} \ N_m \ \xi_\alpha \ e^{-\alpha\phi} \ H^{m\alpha\alpha_3 \dots \alpha_{p+1}}.$$

Further, we take into account symplectic $(n-1)$-form for the potential $B_{\nu_1 \dots \nu_p}$

$$\int_{S_{\mathcal{H}}} \xi^{j_1} \ \Theta^B_{j_1 \dots j_{n-1}} = - \int_{S_{\mathcal{H}}} \epsilon_{\mu j_1 \dots j_{n-1}} e^{-\alpha\phi} H^{\mu\nu_2 \dots \nu_{p+1}} \ \delta B_{\nu_2 \dots \nu_{p+1}}. \tag{16}$$

Using the fact that on the event horizon of black ring $H_{\mu\mu_2 \dots \mu_{p+1}} \xi^\mu \sim \xi_{\mu_2} \dots \xi_{\mu_{p+1}}$ and expressing $\epsilon_{\mu a j_1 \dots j_{n-2}}$ in the same form as in the above case, one arrives at the following:

$$\int_{S_{\mathcal{H}}} \xi^{j_1} \ \Theta^B_{j_1 \dots j_{n-1}} = \frac{p}{(p+1)!} \int_{S_{\mathcal{H}}} \epsilon_{j_1 \dots j_{n-2}} \ e^{-\alpha\phi} \ \xi_\alpha \ H^{\delta\alpha\nu_3 \dots \nu_{p+1}} \ N_\delta \ \xi^{\nu_2} \ \delta B_{\nu_2 \dots \nu_{p+1}}. \tag{17}$$

Combining Eq.(15) with the expressions (17) we finally conclude

$$\bar{\delta} \int_{S_{\mathcal{H}}} Q^B_{j_1 \dots j_{n-2}}(\xi) - \xi^{j_1} \ \Theta^B_{j_1 \dots j_{n-1}} = \Phi_l \ \delta q_l. \tag{18}$$

Let us turn our attention to the contribution connected with gravitational field. Namely, we begin with $Q^{GR}_{j_1 \dots j_{n-2}}(\xi)$ and following the standard procedure [13] one arrives at the following:

$$\int_{S_{\mathcal{H}}} Q^{GR}_{j_1 \dots j_{n-2}}(\xi) = 2\kappa A, \tag{19}$$

where $A = \int_{S_{\mathcal{H}}} \epsilon_{j_1 \dots j_{n-2}}$ is the area of the black ring horizon. It now follows that

$$\bar{\delta} \int_{S_{\mathcal{H}}} Q^{GR}_{j_1 \dots j_{n-2}}(\xi) = 2\delta\biggl(\kappa A\biggr) + 2 \sum_i \delta\Omega_{(i)} \ J^{(i)}, \tag{20}$$

where $J^{(i)} = \frac{1}{2} \int_{S_{\mathcal{H}}} \epsilon_{j_1 \dots j_{n-2} ab} \nabla^a \phi^{(i)b}$ is the angular momentum connected with the Killing vector field $\phi_{(i)}$ responsible for the rotation in the adequate directions. Having in mind calculations conducted in Ref. [13] it could be verified that the following integral is satisfied:

$$\int_{S_{\mathcal{H}}} \xi^{j_1} \ Q^{GR}_{j_1 \dots j_{n-2}}(\xi) = 2A \ \delta\kappa + 2 \sum_i \delta\Omega_{(i)} \ J^{(i)}. \tag{21}$$

The above yields the conclusion that

$$\bar{\delta} \int_{S_{\mathcal{H}}} Q^{GR}_{j_1 \dots j_{n-2}}(\xi) - \xi^{j_1} \ \Theta^{GR}_{j_1 \dots j_{n-1}} = 2\kappa \ \delta A. \tag{22}$$

On using Eqs.(18) and (22), we find that we have obtained the first law of black rings mechanics in Einstein n-dimensional gravity with additional $(p+1)$-form field strength and dilaton fields, in theory which is the simplest generalization of five-dimensional one containing a stationary black ring solution with dipole charge. The first law of black ring mechanics yields the following:

$$\alpha\ \delta M - \sum_i \Omega_{(i)} \delta J^{(i)} + \Phi_l\ \delta q_l = 2\kappa\ \delta A. \tag{23}$$

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